

May 5, 2005
Far Brook School

Instructor: Makoto Yoshida¹

Title of the Lesson: How many circles are there? (Importance of mathematical expressions)

Goal of the Lesson:

1. Help students understand that writing mathematical expressions is useful for communicating and understanding ways of thinking for solving problems.
2. To reinforce students' understanding of the concept of multiplication.
3. Provide opportunities for students to recognize the importance of presenting, listening, and discussing solution ideas in order to improve problem solving skills and learning.

Lesson Background:

One of the important topics that students learn in the 3rd grade is multiplication. Multiplication concepts are typically introduced toward the end of the 2nd grade and student understanding of multiplication concepts and skills to carry out the calculation is enhanced in the 3rd grade. As a part of learning multiplication, students have also learned how to carry out calculations using algorithms (e.g. vertically formatted calculation methods).

Algorithms are very useful in order to line up place values and carry out calculation operations easily and accurately. Therefore, in many textbooks more stress is usually put on learning how to carry out algorithms rather than writing down mathematical expressions. I believe that learning how to write down mathematical expressions is equally important as learning about algorithms.

In middle school and high school mathematics, particularly in algebra, students learn many operation rules such as commutative properties, associative properties, distributive properties, etc. However, by the time they learn these rules they are already dealing with real numbers such as fractions and decimal numbers; positive and negative numbers, and variables such as x and y . Therefore, students need to deal with so many topics all at once. This may contribute to the students feeling that mathematics is a difficult subject and lead them to dislike learning mathematics.

For these reasons, I believe that students should learn how to write mathematical expressions and use them to show their work while they are dealing with easier numbers such as whole numbers. In addition, by learning to communicate ideas through writing down mathematical expressions and by

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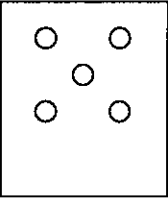
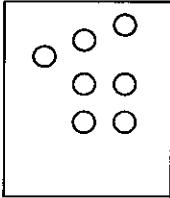
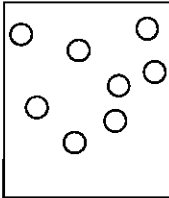
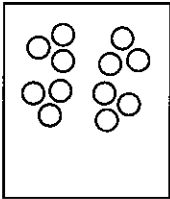
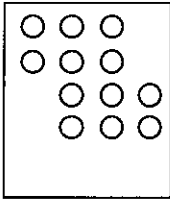
manipulating them to find answers, students can find and learn the nature/rules/properties of calculation operations.

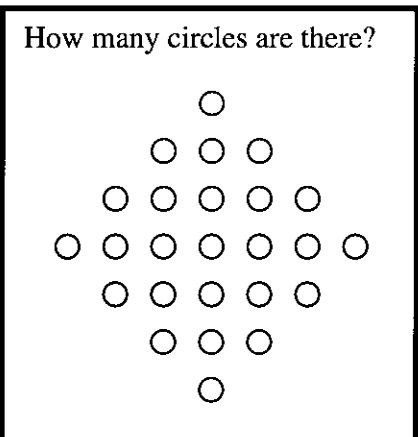
By becoming accustomed to writing down mathematical expressions, students will achieve the following:

- ❖ Proficiency in showing ideas and solution processes
- ❖ Ability to check their solution processes
- ❖ Improvement in communication with others (students and teachers) using mathematical expressions. (Mathematical expressions are like the language of mathematics) Students can learn to express their ideas and way of thinking through mathematical expressions.
- ❖ Ability to manipulate (e.g., combine two math expressions into one math expression to show process and simplify expressions.)
- ❖ Knowledge that mastering the skills to write down mathematical expressions and communicate with others are important skills to have in order to learn more advanced mathematics topics.

In order to further develop this idea, I created a lesson where students are requested to show their solutions using mathematical expressions and diagrams. Having expressions and diagrams together, I also believe that students can solidify understanding of expressions and meaning of numbers used. Through this lesson, I hope students will recognize the importance of mathematical expressions for their learning.

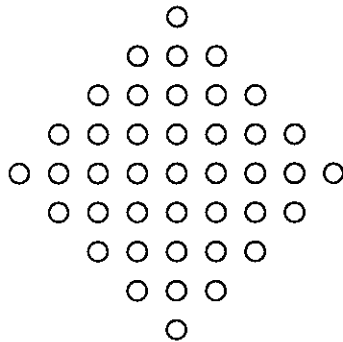
Lesson Process:

Learning Activities, Teacher's Questions, and Expected Student Reactions	Teacher's Support	Points of Evaluation
<p>1. Warm Up Problem</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;">How many circles are there?</div> <p>Show several cards that have circles on them. And ask the students "How many circles are there on the card?"</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="border: 1px solid black; padding: 10px; margin: 5px;">  </div> <div style="border: 1px solid black; padding: 10px; margin: 5px;">  </div> <div style="border: 1px solid black; padding: 10px; margin: 5px;">  </div> </div> <div style="display: flex; justify-content: space-around; align-items: flex-start; margin-top: 20px;"> <div style="border: 1px solid black; padding: 10px; margin: 5px;">  </div> <div style="border: 1px solid black; padding: 10px; margin: 5px;">  </div> </div>	<p>Try to capture many different student ideas. Write down their ideas on the board. Ask students how we can show those ideas with math sentences. Compare those ideas and discuss merits of using one method over another.</p> <p>Point out:</p> <ul style="list-style-type: none"> ❖ Ideas can be express with math sentences ❖ There are a variety of ideas to think about ❖ Making groups can be helpful. 	<p>Are they interested in the task?</p> <p>Can they share their ideas with the class?</p> <p>Can they come up with math sentences for associated ideas?</p> <p>Can they recognize some methods are better than others?</p>

<p>2. Introduction to the Main Problem</p> <div style="border: 1px solid black; padding: 10px; text-align: center;"> <p>How many circles are there?</p>  </div> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; margin-top: 10px; width: fit-content;"> <p>Please think about using multiplication to solve this problem.</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 5px; margin-top: 10px; width: fit-content;"> <p>Please write a mathematical expression to show how you solved it.</p> </div>	<p>Make sure that the students understand they need to use multiplication to solve this problem.</p>	<p>Did they understand the problem? Did they clearly understand the directions?</p>
<p>3. Solving the Problem Individually</p> <p>Examples of students' anticipated responses (see attached sheet)</p>	<p>Walk around the classroom to encourage the students to write math sentences. Also identify various solution methods.</p> <p>Pass out worksheet for presentation and explain how to use it.</p>	
<p>4. Student Presentation</p> <p>Students will present their solutions using presentation worksheet and mathematical expressions.</p> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; margin-top: 20px; width: fit-content;"> <p>Which one of the solutions do you think is better? Why do you think so?</p> </div>	<p>Help all students in the classroom to understand each solution. Make sure to write math sentences on the board.</p> <p>Find out similarities and differences in methods.</p> <p>Point out:</p> <ul style="list-style-type: none"> ❖ Ideas can be expressed with math sentences ❖ Variety of ideas but some of them are better than others. ❖ Making groups can be helpful. 	<p>Are they interested in listening to other students' presentations?</p> <p>Can they understand the differences among different methods?</p>

5. Solving Another Problem (Optional)

How many circles are there?



What happened to the circles of this problem?

S: More circles.

S: There are 5 circles outside of the diamond.
(Last one was 4 circles.)

What are the similarities and differences between these problems?

I wonder if any of the solutions we found before can be used to solve this problem. What do you think?

Just like the last time, please think about using multiplication to solve this problem. And please write a mathematical expression to show how you solved it.

Make sure to talk about similarities and differences of these two problems.

Are they interested in the problem?

Were they able to participate in the discussions?

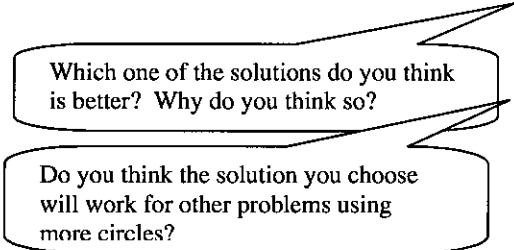
Make sure that the students understand they need to use multiplication to solve this problem.

Did they understand the problem? Did they clearly understand the directions?

6. Solving the Problem Individually (Optional)

Examples of students' anticipated responses
(see attached sheet)

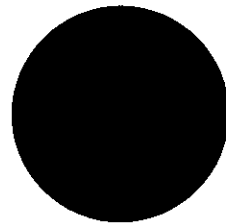
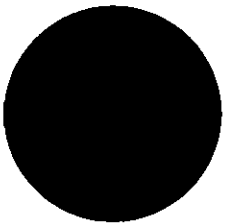
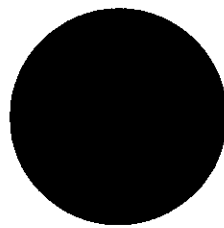
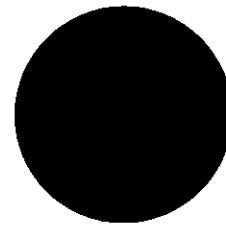
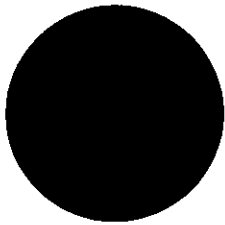
Walk around the classroom to encourage the students to write mathematical expressions. Also identify various solution methods.

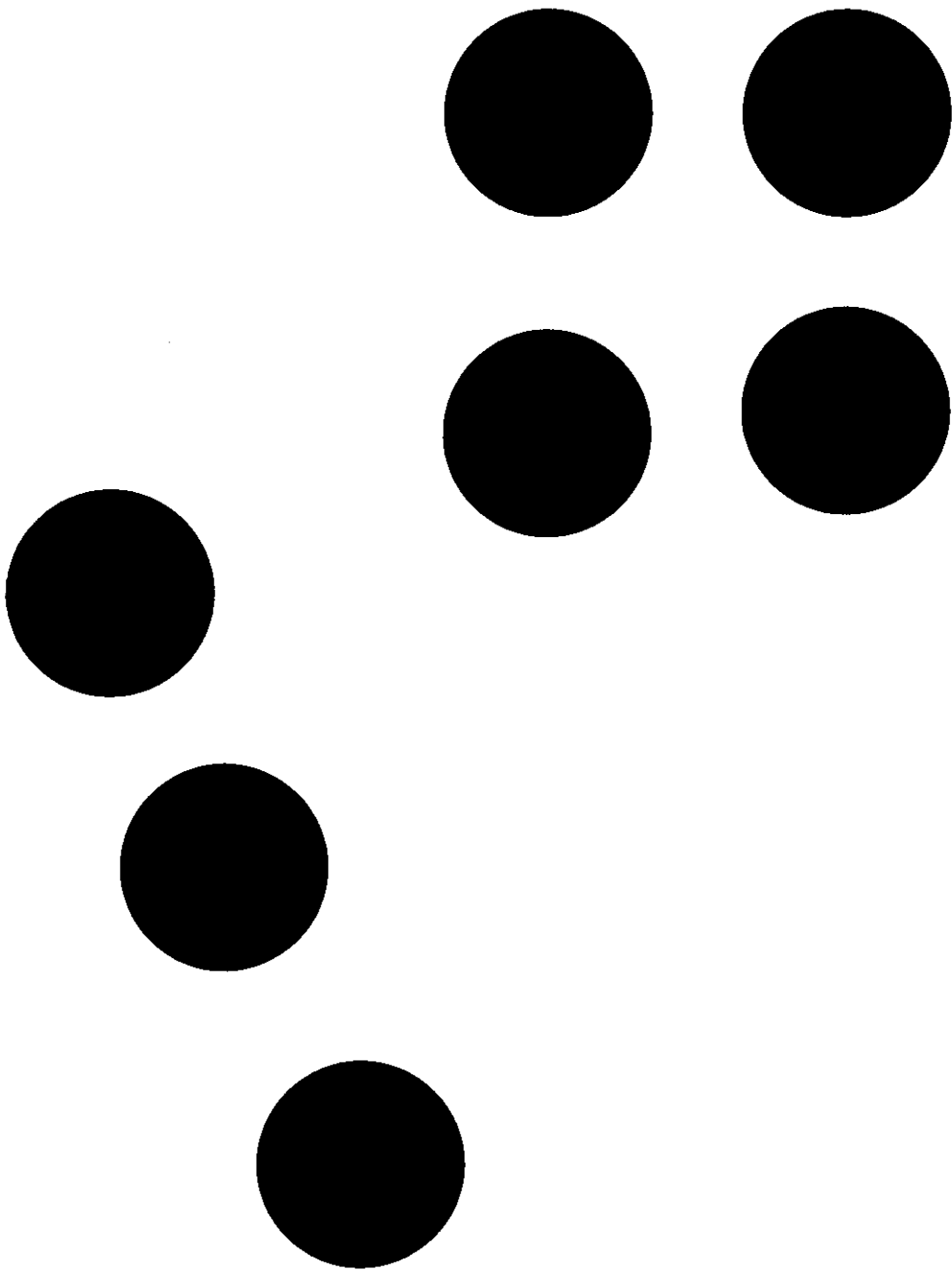
<p>7. Student Presentation (Optional) Students will present their solutions using presentation worksheet and mathematical expressions.</p> 	<p>Pass out worksheet for presentation.</p> <p>Point out:</p> <ul style="list-style-type: none"> ❖ Ideas can be expressed with math sentences ❖ Variety of ideas but some of them are better than others. ❖ Making groups can be helpful. ❖ Some solutions that students found in problem one do not work with this problem 	
<p>7. Conclusion</p>	<p>Summarize the lesson and highlight the importance of writing mathematical expressions. Also, talk about the importance of listening to each other's ideas in order to learn other solution ideas.</p>	

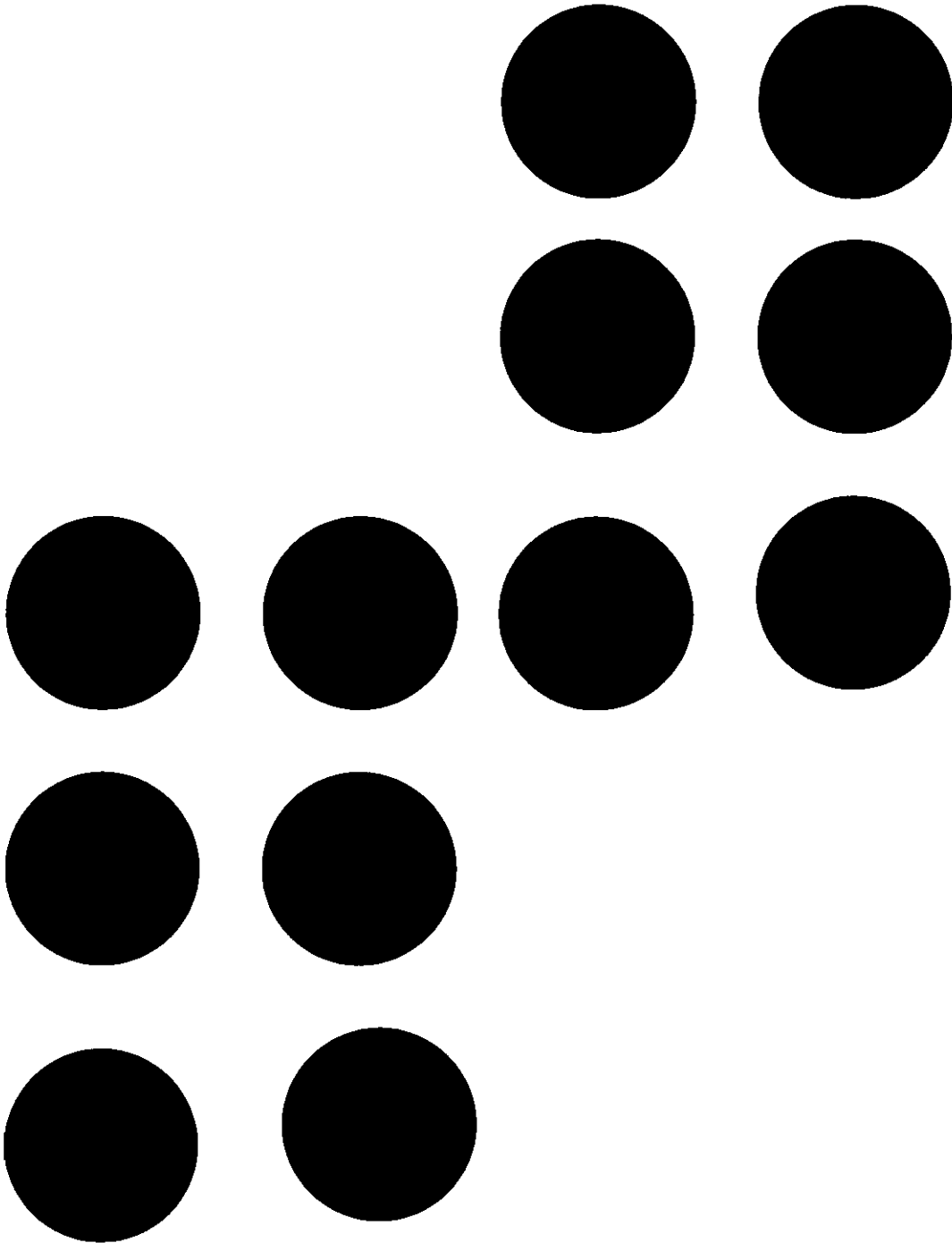
Evaluation of the Lesson:

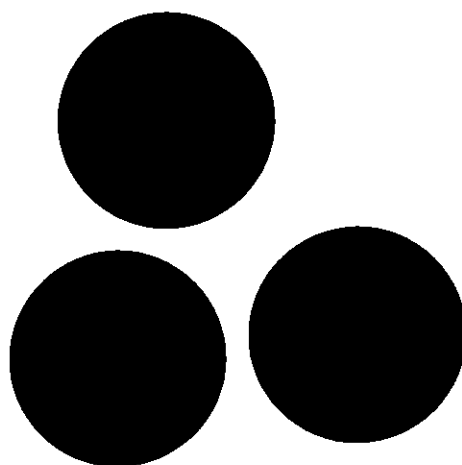
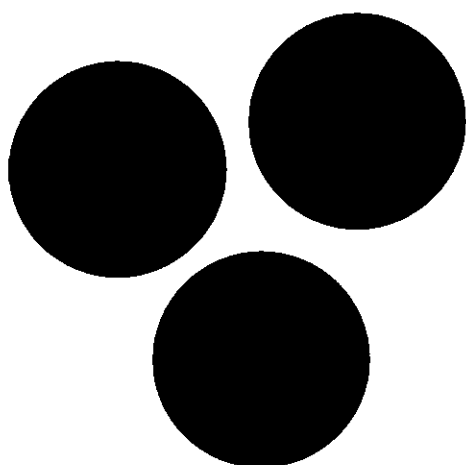
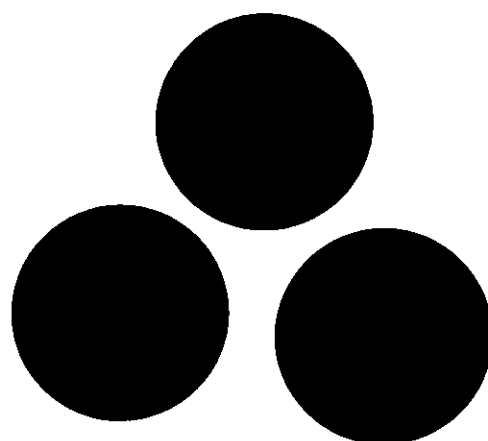
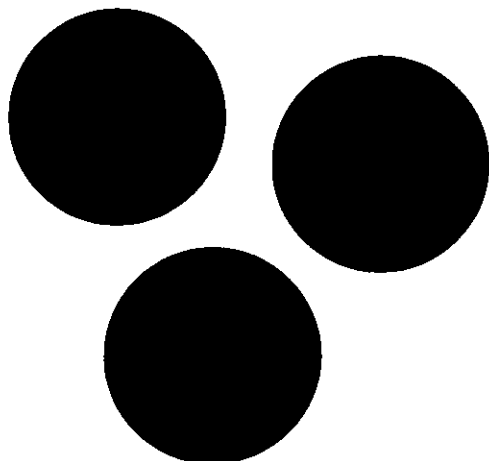
1. Did the students understand that writing mathematical expressions is useful for communicating and understanding ways of thinking for solving problems?
2. Did they use multiplication to solve the problems? Do they understand the concept of multiplication?
3. Did they work together with their classmates? Were they thinking and influencing each other for deeper understanding and learning?

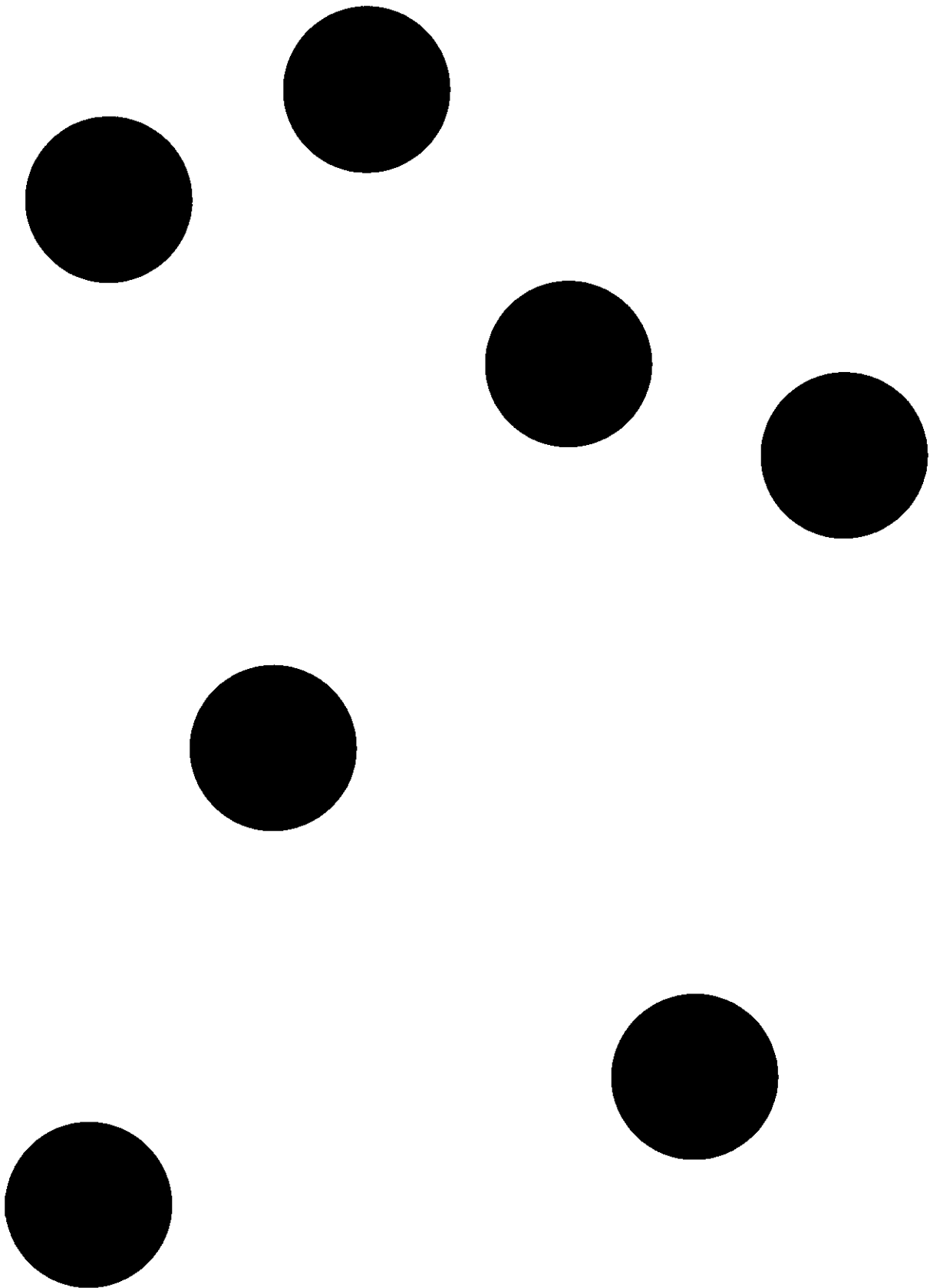
How many
circles are
there?











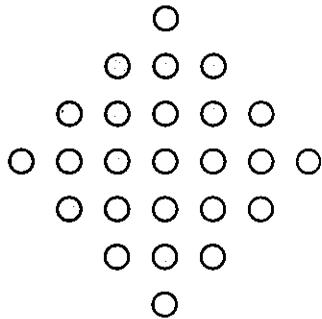
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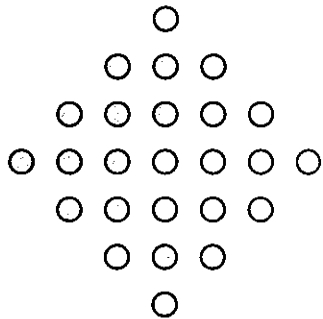
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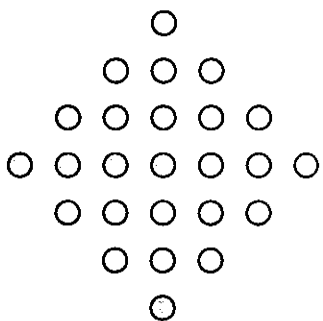
Problem No. 1

How many circles are there?

Please think about using multiplication to solve this problem and write a math sentence to show how you solved it.







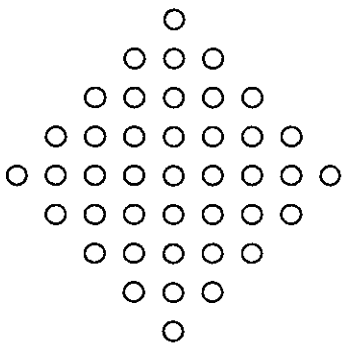
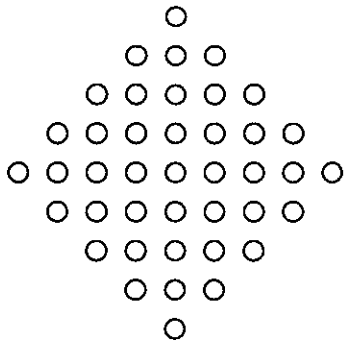
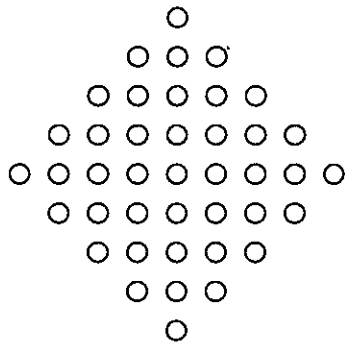
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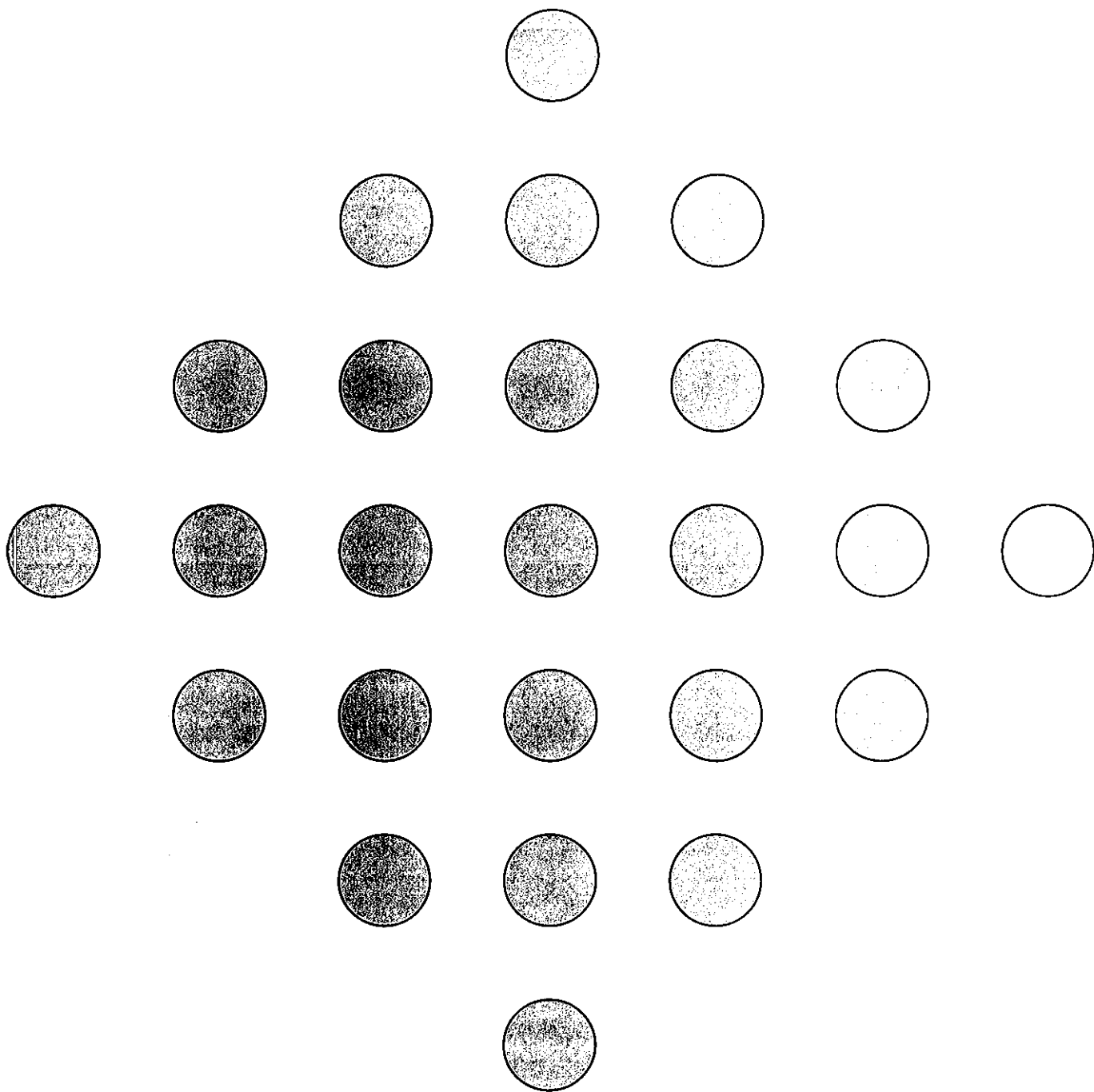
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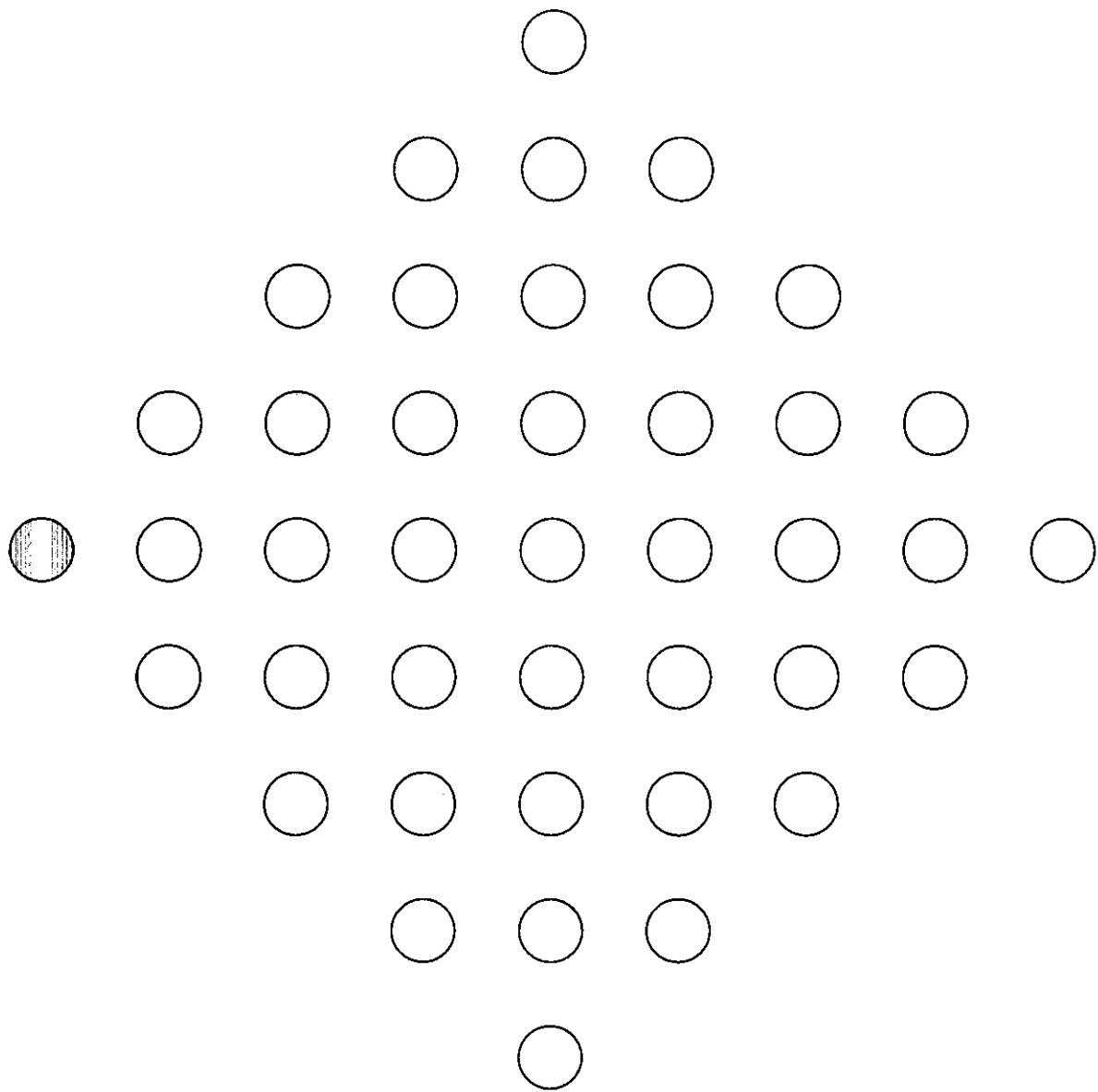
Problem No. 2

How many circles are there?

Please think about using multiplication to solve this problem and write a math sentence to show how you solved it.

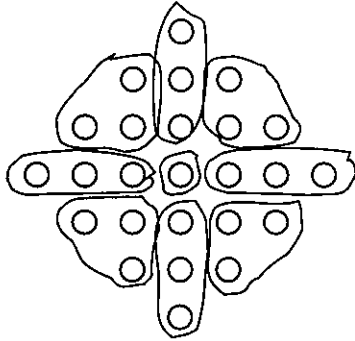




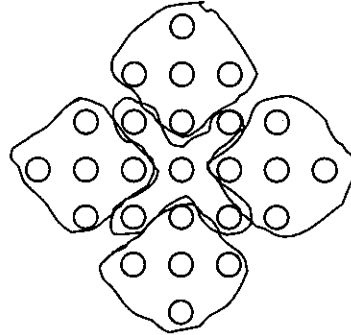


Examples of Students' Anticipated Solutions

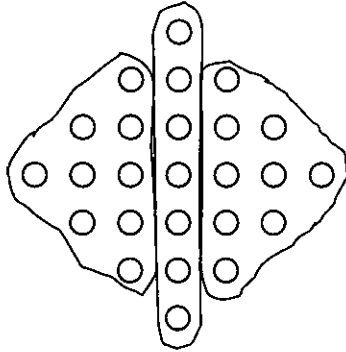
Problem No. 1



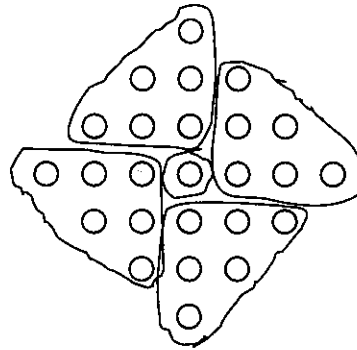
$$8 \times 3 + 1 = 25$$



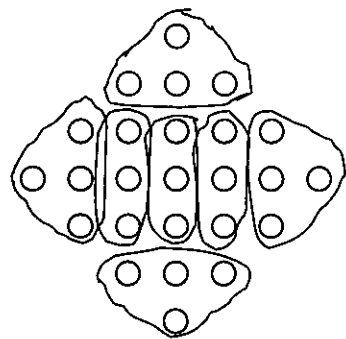
$$5 \times 5 = 25$$



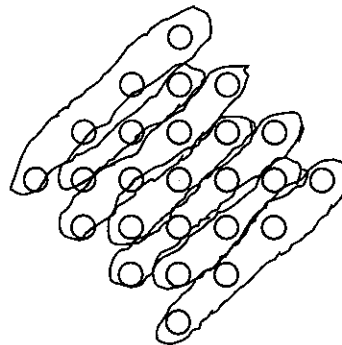
$$2 \times 9 + 7 = 25$$



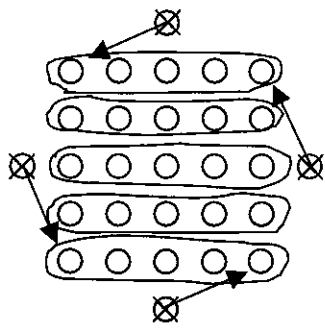
$$4 \times 6 + 1 = 25$$



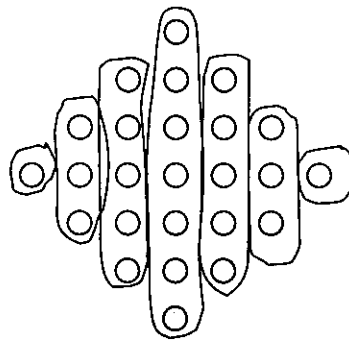
$$4 \times 4 + 3 \times 3 = 25$$



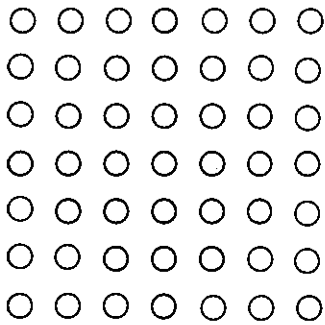
$$4 \times 4 + 3 \times 3 = 25$$



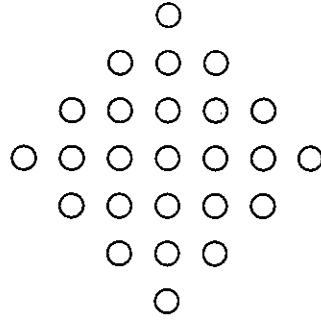
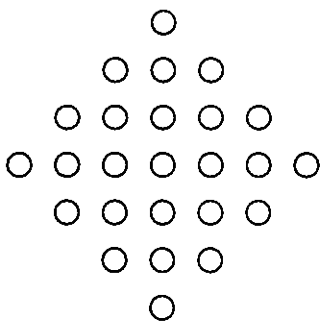
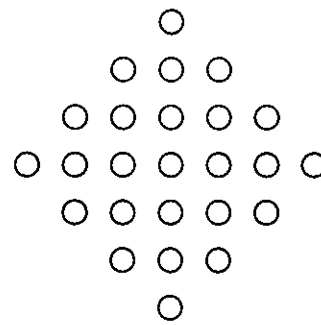
$$5 \times 5 = 25$$



$$7 + 2 \times 5 + 2 \times 3 + 2 \times 1 = 25$$

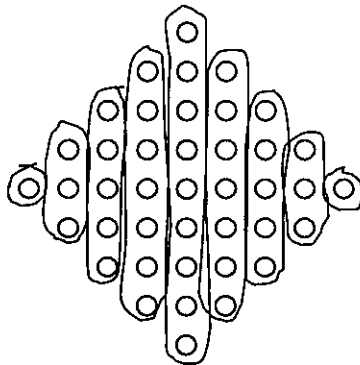


$$7 \times 7 - 4 \times 6 = 25$$

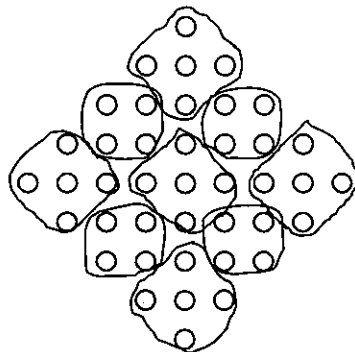


Examples of Students' Anticipated Solutions

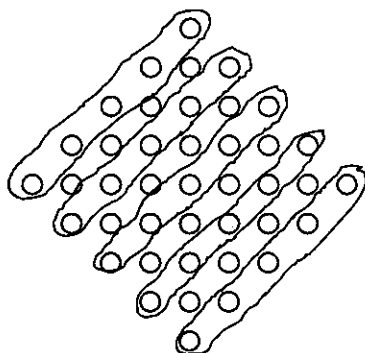
Problem No. 2



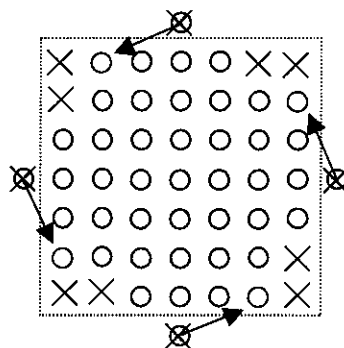
$$\begin{aligned} &9 + 2 \times 7 + 2 \times 5 + 2 \times 3 + 2 \times 1 \\ &= 9 + 14 + 10 + 6 + 2 \\ &= 41 \end{aligned}$$



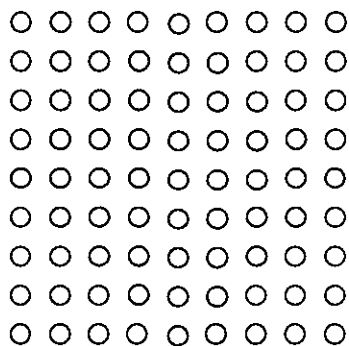
$$5 \times 5 + 4 \times 4 = 41$$



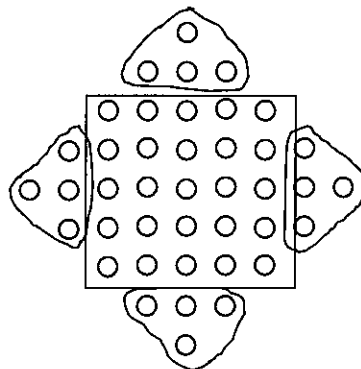
$$5 \times 5 + 4 \times 4 = 41$$



$$7 \times 7 - 4 \times 2 = 41$$



$$9 \times 9 - 4 \times 10 = 41$$



$$5 \times 5 - 4 \times 4 = 41$$